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LETTERS TO THE EDITOR

Dear Editor,

On losses in $M^X/GI/1/n$ queues

The goal of this letter is to improve the recent result of Righter (1999) on losses in $M^X/GI/1/n$ queues.

The brief history of the question is the following. Consider an $M/GI/1/n$ queueing system in which the expectations of interarrival and service times are equal to each other, where n is the number of spaces in the queue excluding the space for the customer in service. Abramov (1991a), (1997) and Righter (1999) proved that the expectation of the number of losses during a busy period is equal to 1 for all $n \geq 0$. Righter (1999) found a relationship between L_{n+1} and L_n, where L_n is the number of lost customers during a busy period of an $M/GI/1/n$ queueing system. It was shown for the $M^X/GI/1/n$ queueing system that, if $\lambda\beta = \mu$, where λ is the arrival rate, μ is the reciprocal of the expectation of service time, and β is the expectation of batch size, then $\mathrm{E}\, L_n \leq 1$ for all $n \geq 0$, provided that a busy period starts from a single customer. This result follows by developing the relationship obtained in the case of ordinary (nonbatch) arrivals and is intuitively clear from the author's explanation based on the stochastic order relation. For other relevant applications related to the $M/GI/1/n$ and $GI/M/1/n$ queues see also the papers of Peköz (1999) and Abramov (1991b), (2001).

In this letter we give a simple proof that the property $\mathrm{E}\, L_n = 1$ for all $n \geq 0$ holds also for the case of the $M^X/GI/1/n$ queueing system, thus improving the result of Righter (1999).

Consider the $M^X/GI/1/n$ queueing system. Let Y denote the number of customers in the batch that starts a busy period, let T_n denote a busy period, and let ν_n and L_n be the numbers of served and lost customers during a busy period respectively.

Proposition 1. *For the $M^X/GI/1/n$ queueing system under the assumption that $\lambda\beta = \mu$, we have* $\mathrm{E}\{L_n \mid Y\} = Y$.

Proof. Applying the renewal reward theorem, we obtain

$$\mathrm{E}\{\nu_n \mid Y = k\} + \mathrm{E}\{L_n \mid Y = k\} = \lambda\beta\, \mathrm{E}\{T_n \mid Y = k\} + k, \tag{1}$$

$$\mathrm{E}\{\nu_n \mid Y = k\} = \mu\, \mathrm{E}\{T_n \mid Y = k\}. \tag{2}$$

The sense of equation (1) is the following. The left-hand side denotes the expected number of arrived customers during a regeneration period, and the term $\lambda\beta\, \mathrm{E}\{T_n \mid Y = k\}$ on the right-hand side denotes the expected number of arrived customers during a regeneration period excluding the first k customers of the first batch. Then, keeping in mind that $\lambda\beta = \mu$, from (1) and (2) we obtain

$$\mathrm{E}\{L_n \mid Y = k\} = k, \tag{3}$$

for all $n \geq 0$ and $k \geq 1$, and the desired statement obviously follows from (3).

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References

ABRAMOV, V. M. (1991a). *Investigation of a Queueing System with Service Depending on Queue Length*. Donish, Dushanbe (in Russian).

ABRAMOV, V. M. (1991b). Asymptotic properties of lost customers for one queueing system with refusals. *Kibernetika* **2,** 123–124 (in Russian).

ABRAMOV, V. M. (1997). On a property of a refusals stream. *J. Appl. Prob.* **34,** 800–805.

ABRAMOV, V. M. (2001). Inequalities for the $GI/M/1/n$ loss system. *J. Appl. Prob.* **38,** 232–234.

PEKÖZ, E. A. (1999). On the number of refusals in a busy period. *Prob. Eng. Inf. Sci.* **13,** 71–74.

RIGHTER, R. (1999). A note on losses in $M/GI/1/n$ queues. *J. Appl. Prob.* **36,** 1240–1243.

Yours sincerely,

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