Access and service rate control in queuing system ⋆

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Abstract: The problem of access and service rate control as a general optimization problem for controlled Markov process with finite state space is considered. By using the dynamic programming approach we obtain the explicit form of the optimal control in the case of minimizing cost given as a mixture of an average queue length, number of lost jobs, and service resources. The problem is considered on a finite time interval in the case of non stationary input flow. In this case we suggest the general procedure of the numerical solution which can be applied to a problems with constraints.

1. INTRODUCTION

Effective control of data flows is one of the most important problems of the Internet of the next generation, which conform both to new realities of super speed networks (according to the present concepts) with integral servicing. Even though there exists a number of empirical and engineering approach, the theoretical basis of data transmission control is still restricted by stationary methods of analysis. Moreover, most existing approaches are based on asymptotic methods, which give only a qualitative description of network behavior, but do not correspond to real-time control and design of real network control algorithms, like various Active Queue Management (AQM) schemes (see Vasenin & Simonova [2005]).

Here we consider the non-stationary feedback type controls. These problems are inherent to queuing systems on finite horizon under restricted control resources. We underline that these problem are different from usually considered ones in infinite time interval (see, for example Hordijk & Spiekma [1989], Kelly et al. [1998], Low et al. [2002] Piunovskiy [2004], Serfozo [1981]). First, in the case of infinite horizon one have to provide the stability, in other words the service rate must be greater or equal to the rate of demands arriving. Another disadvantage concerns the necessity of stationary data which is not a case for real service systems. Moreover, the optimization of a stationary phase does not take into account the cost of transient phases and resources which are needed for their realization.

One of the most general problem statement is given in Hordijk & Spiekma [1989], where a queueing system can be controlled by restricting arrivals. Different settings of optimization problems related with stochastic networks are given in Kelly et al. [1998], where the approach to solution is also based on optimization techniques of convex mathematical programming. In Piunovskiy [2004] the problem of the input stream control arising in communication networks is also considered and reduced to a convex programming problem. It is worth to underline that in the problems with long-run average type criteria and stationary controls the optimal solution is very often has a threshold form.

Meanwhile, in the case of finite horizon the threshold type controls are inherent to settings with affine dependence on control action. In control of communication networks this result has been obtained, probably first in Bremaud [1979], where the problem of optimal thinning of a point process has been solved in the case of non-stationary input flow with deterministic intensity rate.

General approach to these problems is based on the martingale description of the process evolution (see Bremaud [1981], Elliott et al. [1995], Liptser & Shiryaev [2005]). The existence of the optimal solution had been proved in in Davis & Elliott [1977] Wan & Davis [1979]. In Elliott [1992], Elliott et al. [1995] the general optimization problem for jump Markov process had been considered and the reduction to a problem with complete information had been proposed for a wide class of optimal control problems. In Miller et al. [2005] we extend this approach to a flow control with state-control dependent rate.

In this article we consider some typical problems of network control in non stationary case. We extend the approach of Bremaud [1979] to more wide class of the optimal control problems with complete information and prove the existence and characterization of the optimal control with the aid of dynamic programming approach. In this case the dynamic programming equation can be reduced to the system of ordinary differential equations. Then we apply these result to a problem of access and service rate control in the case of finite time horizon and finite buffer size.

We demonstrate that for a wide class of criteria the optimal control problem can be reduced to the solution of the system of ordinary differential equations. Moreover, the optimal control exists within the class of Markov strategies and therefore can be calculated in "program

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form" for each possible state of controlled Markov chain. The structure of the article is as follows. In the next section we provide some necessary results from the theory of controlled Markov chains. In Section III apply these results to the simultaneous access and service rate control. In Section IV we give some examples.

2. CONTROLLED MARKOV CHAIN

In this section we extend the approach of P. Bremaud (see Bremaud [1979]) to a more general class of controlled Markov chains basing on their martingale description (see Aggoun & Elliott [2004], Elliott et al. [1995]). This section presents a slight generalization of well-known result of Bruce Miller [1968] who considered the controls taking values in a finite set. It had been shown that the optimal control exists within the class of piece-wise constant policies. Generally it is not always true for an arbitrary class of cost functions, however, for the most examples arising in network optimization it is still valid.

2.1 Martingale representation of controlled Markov chain

Assume that all processes are defined on a probability space $\{\Omega, \mathcal{F}, P\}$. Consider a process $\{X_t,\ t \in [0,T]\}$ which is a controlled jump Markov process with piecewise constant right-continuous trajectories. The state space of the process is the set of unit vectors $e_i \in \mathbb{R}^n$: $X_t \in S = \{e_1, \dots, e_n\}$.

Assumption 2.1. The matrix $A(t,u)$ with elements $a_{ij}(t,u)$ is a time-dependent family of generators, such that the probability column vector $p_t = (p_t^1, ..., p_t^n)^*$, where $p_t^i = P(X_t = e^i)$ satisfies the Kolmogorov forward equation

$$\frac{dp_t}{dt} = A(t,u)p_t. \tag{1}$$

Here the control parameter $u \in U$, where U is some compact set in complete metric space and $A(t,u)$ is continuous on $[0,T] \times U$.

Introduce the following right-continuous sets of complete σ-algebras generated by X_t

$$\mathcal{F}_t^X = \sigma\{X_s :\ s \in [0,t]\}.$$

Assumption 2.2. We assume that the set $\underline{U}$ of admissible controls $\{u(\cdot)\}$ is the set of $\mathcal{F}_t^X$-predictable processes with values in U. This means that, if N_t is the number of the state changes, X_0^t is the series of states occurred from the origin at $t = 0$ until the current time $t \in [0,T]$, that is

$$X_0^t = \{(X_0, 0), (X_{\tau_1}, \tau_1), \dots, (X_{\tau_{N_t}}, \tau_{N_t})\}$$

is the set of states and jump times, then for $\tau_{N_t} < t \le \tau_{N_t+1}$ the control $u_t = u(t, X_0^t)$ is a function of X_0^t and the current time t (see Bremaud [1981], Elliott et al. [1995]).

For each control function $u(\cdot) \in \underline{U}$ the process $\{X_t\}$ satisfies the following system of stochastic differential equations:

$$X_t = X_0 + \int_0^t A(s,u_s)X_s\, ds + M_t, \tag{2}$$

where X_0 is the initial condition, and $M_t := \{M_t^1, \dots, M_t^n\}$ is a square integrable $(\mathcal{F}_t^X, P)$ martingales with the following quadratic variations [1] (see Bremaud [1981], Elliott et al. [1995], Liptser & Shiryaev [2005]):

$$\left\langle M \right\rangle_t = -\int_0^t \left[A(s,u_s)(diag\ X_s) + (diag\ X_s)A^*(s,u_s)\right] ds + \int_0^t diag\ (A(s,u_s)X_s)\ ds, \tag{3}$$

where $diag\ X$ denotes the matrix with diagonal entries $X^1, \dots, X^n$ and A^* denotes the transposed matrix of A.

Remark 2.3. In other words process $X(t)$ is a solution of *martingale problem* (2),(3) for controlled Markov chain (see Elliott et al. [1995]).

2.2 Performance criterion

The optimization goal is to minimize some cost function of the Markov chain states and controls. This function could take into account the average queue length, which is related with the average time of service, or/and the price of rejected (thinned) demands, since they have to either repeatedly queue or choose another service center. Moreover, in the case of finite time horizon the final state of Markov chain is also very important, like in the case of congestion resolution. So we consider the following performance criterion

$$J[u(\cdot)] = \mathbf{E}\left\{\phi_0(X_T) + \int_0^T f_0(s,u_s,X_s)ds\right\} \to \min_{u(\cdot)}, \tag{4}$$

with

$$\phi_0(X) = \langle \phi_0, X\rangle, \qquad f_0(s,u,X) = \langle f_0(s,u), X\rangle,$$

where $\langle \cdot, \cdot \rangle$ is a sign of scalar product and

$$\phi_0 \in R^n,$$
$$f_0^*(s,u) = (f_0(s,u,e_1), \dots, f_0(s,u,e_n))$$

and each $f_0(s,\cdot,e_i)$ is a cost function when the Markov chain is in state e_i at time $s \in [0,T]$.

Assumption 2.4. Each of functions $f_0(s,\cdot,e_i)$ is continuous on $[0,T] \times U$ and bounded below.

2.3 Dynamic programming and optimal control

Define the value function

$$V(t,X) = \inf_{u(\cdot)} J[u(\cdot)|X_t = X], \tag{5}$$

where

$$J[u(\cdot)|X_t = X] = \mathbf{E}\left\{\phi_0(X_T) + \int_t^T f_0(s,u_s,X_s)ds \,\middle|\, X_t = X\right\}. \tag{6}$$

Moreover, according to Assumption 2.4 the infimum in (5) exists, and function $V(t,X)$ admits the representation

$$V(t,X) = \langle \phi(t), X\rangle,$$

with some continuous vector-valued function $\phi(t) = (\phi^1(t), ..., \phi^n(t))^* \in R^n$.

[1] $\left\langle M \right\rangle_t$ is the quadratic variation of a martingale M, and $\left\langle M, N \right\rangle_t$ is the mutual quadratic variation of martingales M_t and N_t.

Consider the following equation (*dynamic programming equation*) with respect to vector-valued function $\phi(t)$

$$\langle \phi'(t), X\rangle + \min_{u\in U} [\langle \phi(t), A(t,u)X\rangle + \langle f_0(t,u), X\rangle] = 0, \quad (7)$$

with terminal condition

$$\phi(T) = \phi_0.$$

Since the function

$$H(\phi, t, u, X) = \langle \phi, A(t,u)X\rangle + \langle f_0(t,u), X\rangle,$$

is continuous with respect to (t, u) and affine with respect to ϕ, then for any $(t, X) \in [0, T] \times S$ function

$$\mathcal{H}(\phi, t, X) = \min_{u\in U} H(\phi, t, u, X)$$

is Lipshitz in ϕ. Next result follows immediately from above considerations.

Proposition 1. Let the Assumptions 2.1, 2.4 hold. Then the equation (7) has the unique solution on $[0, T]$.

Remark 2.5. Equation (7) can be written as a system of ordinary differential equations

$$\frac{d\phi^i(t)}{dt} = -\mathcal{H}(\phi(t), t, e_i), \quad i = 1, ..., n \quad (8)$$

which can be obtained by substituting $X = e_i, \quad i = 1, ..., n$.

Theorem 2.6. Assume that:

$\phi(t)$ is the solution of system (8) and there exists $u_0(t, X) \in U$ such that at each $(t, X) \in [0, T] \times S$ the value on the right-hand side of (7) and function $H(\phi(t), t, u, X)$ achieves the minimum at $u_0(t, X)$.

Then there exists $\hat{u}(t, X_0^t)$ in the class of $\mathcal{F}_t^X$ predictable controls which is the optimal control and $V(t, X) = J[\hat{u}(\cdot)|X_t = X]$.

3. ACCESS AND SERVICE RATE CONTROL MODEL

We consider a queueing system that can be controlled by restricting arrivals and by changing of the service rate. We assume that the jobs flow constitute a counting process with deterministic rate $\lambda(t) \geq 0$. The number of jobs in the system is bounded by some constant $N < \infty$ and the service rate is $\mu \in [\underline{\mu}, \overline{\mu}]$, where $\underline{\mu} > 0$. Control $u(t) \in [0, 1]$ is a probability to accept the job at time $t \in [0, T]$. So the part of arriving jobs can be rejected and the performance criterion takes into account the number of rejected jobs and the average queue time for the accepted jobs. Our model is motivated by Hordijk & Spiekma [1989], where this problem is considered in infinite time horizon in the class of stationary controls of *threshold* or *thinning* types.

3.1 Controlled Markov chain model

Assume that the state X is a number of jobs in the system, so the number of states is $N + 1$, and the corresponding state space S consists of vectors $\{e_0, ..., e_N\}$.

Proposition 2. $(N + 1) \times (N + 1)$ matrix $A(t, u, \mu)$ has a form

$$A(t,u,\mu) = \begin{pmatrix} -\lambda(t)u & \mu & 0 & \dots & 0 & 0 & 0 \\ \lambda(t)u & -\mu-\lambda(t)u & \mu & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \lambda(t)u & -\mu-\lambda(t)u & \mu \\ 0 & 0 & 0 & \dots & 0 & \lambda(t)u & -\mu \end{pmatrix}, \quad (9)$$

where control $(u, \mu) \in [0, 1] \times [\underline{\mu}, \overline{\mu}]$.

Proof:

Take some $\mathcal{F}_t^X$ - predictable controls $u(t), \mu(t)$. Let $N_t \in \{0, ..., N\}$ is a current number of jobs in the system. This number changes due to the action of two flows: flow of arrivals and departure flow of outcome completed jobs. We assume that the arrival flow forms a counting process with intensity $\lambda(t) \geq 0$, so the number of demands N_t^a arriving to the system from origin at $t = 0$ until current time can be represented as follows Liptser & Shiryaev [2005]

$$N_t^a = \int_0^t \lambda(s)ds + M_t^a,$$

where M_t^a is a square integrable martingale with quadratic variation

$$\left\langle M^a \right\rangle_t = \int_0^t \lambda(s)ds.$$

The departure flow is a counting process with state-dependent rate $\mu(t)I\{N_t > 0\}$, where $I\{\cdot\}$ is an indicator function. Therefore, departure flow N_t^d admits the representation

$$N_t^d = \int_0^t \mu(s)I\{N_s > 0\}ds + M_t^d,$$

where M_t^d is a square integrable martingale with quadratic variation

$$\left\langle M^d \right\rangle_t = \int_0^t \mu(s)I\{N_s > 0\}ds.$$

We suppose, that N_t^a and N_t^d are independent and do not have jumps at the same time, it means that the mutual quadratic variation $\left\langle M^a, M^d \right\rangle_t = 0$.

As shown by P. Bremaud (see Lemma 1 in Bremaud [1979]) the access control can be represented as a control of intensity of the arrival flow. So if $W(t)$ is an access control, that is it is a random variable, taking values in $\{0, 1\}$ and such that controlled arrival flow is equal to

$$N_t^{a,c} = \sum_{\tau\leq t} I\{N_\tau < N\}I\{W(\tau) = 1\}\Delta N_\tau^a,$$

then

$$\begin{aligned} &E\{I\{W(t) = 1\}I\{N_t < N\}|\mathcal{F}_t^X\} \\ &= u(t)I\{N_t < N\} = u(t)I\{X_t \neq e_N\}, \end{aligned} \quad (10)$$

and $u(t) \in [0, 1]$ is $\mathcal{F}_t^X$ predictable process.

Then,

$$\Delta N_t = \Delta N_t^{a,c} - \Delta N_t^d.$$

Taking into account the relation

$$I\{N_t = i\} = I\{X_t = e_i\}$$

one can write

$$\Delta X_t = A^+ X_{t-} \Delta N_t^{a,c} + A^- X_{t-} \Delta N_t^d,$$

where $(N+1)\times(N+1)$ matrices A^+, A^- have a form

$$A^+ = \begin{pmatrix} -1 & 0 & 0 & \dots & 0 & 0 \\ 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix},$$

$$A^- = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ 0 & 0 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 \end{pmatrix}.$$

Then, by using the relation

$$X_t = X_0 + \sum_{\tau \le t} \Delta X_\tau,$$

and taking into account that of $N_t^{a,c}$ and N_t^d are counting processes we obtain

$$X_t = X_0 + \sum_{\tau \le t} [A^+ X_{\tau-} \Delta N_\tau^{a,c} + A^- X_{\tau-} \Delta N_\tau^d]$$
$$= X_0 + \int_0^t A^+ X_{s-} dN_s^{a,c} + \int_0^t A^- X_{s-} dN_s^d.$$

Finally, we have to substitute the martingale representation of $N_t^{a,c}$ and N_t^d into above equation and by taking the conditional expectation with respect to $\mathcal{F}_t^X$ we obtain that

$$X_t = X_0 + \int_0^t [A^+ \lambda(s) u(s) + A^- \mu(s)] X_s ds + M_t^{u,\mu}$$
$$= \int_0^t A(s, u(s), \mu(s)) X_s ds + M_t^{u,\mu},$$

where $M_t^{u,\mu}$ is a square integrable $\mathcal{F}_t^X$ martingale with quadratic variation

$$\left\langle M^{u,\mu} \right\rangle_t$$
$$= \int_0^t [A^+ X_s X_s^* (A^+)^* \lambda(s) u(s) + A^- X_s X_s^* (A^-)^* \mu(s)] ds.$$

Routine calculation shows that this expression coincides with (3).

3.2 The performance criterion

As we mention above the performance criterion takes into account the average time in queue, which can be estimated as follows

$$J_1 = \mathbf{E}\left\{ \int_0^T \frac{N_s}{\mu(s)} ds \right\} = \mathbf{E}\left\{ \int_0^T \frac{\langle \mathbf{l}, X_s \rangle}{\mu(s)} ds, \right\}$$

where

$$\mathbf{l}^* = (0, 1, 2, ..., N) \in R^{N+1}.$$

Another criterion to be minimized is an average number of rejected jobs, which can be calculated with the aid of the integral representation given by Wong and Hajek (see [Wong & Hajek, 1985, p. 261, Lemma 3.2]) and by using the relation (10) as follows

$$J_2 = \mathbf{E}\{N_T^a - N_T^{a,c}\}$$
$$= \mathbf{E}\left\{ \int_0^T [1 - u(\tau)\langle \mathbf{1}, X_\tau \rangle] \lambda(\tau) d\tau \right\},$$

where

$$\mathbf{1}^* = (1, 1, ..., 1, 0) \in R^{N+1}.$$

Third criterion represents the service resources spent during the control interval

$$J_3 = \int_0^T \mu(\tau) \langle \mathbf{I}, X_\tau \rangle d\tau,$$

where

$$\mathbf{I}^* = (0, 1, ..., 1) \in R^{N+1},$$

Further we consider the performance criterion which is a mixture of J_1, J_2 and J_3, that is

$$J = k_1 J_1 + k_2 J_2 + k_3 J_3,$$

where $k_i \ge 0, i = 1..3$.

3.3 Dynamic programming equation and optimal control

So we have to solve the equation (7), where $A(t, u, \mu)$ is defined by (9), and cost function is defined by the function $f_0(t, u, \mu, X)$

$$f_0(t, u, \mu, X) = k_1 \frac{\langle \mathbf{l}, X \rangle}{\mu} + k_2 [1 - u\langle \mathbf{1}, X \rangle] \lambda(t) + k_3 \mu \langle \mathbf{I}, X \rangle.$$

The Hamiltonian $H(\phi, t, u, \mu, X)$ is affine in u, i.e.

$$H(\phi, t, u, X) = H_0(\phi, t, \mu, X) + u H_1(\phi, t, X)$$

then the dynamic programming equation has a form of the following system of ordinary differential equation

$$0 = \frac{d\phi^i(t)}{dt} + \min_{\left(\substack{u \in [0,1] \\ \mu \in [\underline{\mu}, \overline{\mu}]} \right)} [H_0(\phi, t, \mu, e_i) + u H_1(\phi, t, e_i)]$$
$$= \frac{d\phi^i(t)}{dt} + \min_{u \in [0,1]} [\min_{\mu \in [\underline{\mu}, \overline{\mu}]} H_0(\phi, t, \mu, e_i) + u H_1(\phi, t, e_i)]$$

for $i = 0, ..., N$, where,

$$H_0(\phi, t, \mu, e_i) = \mu \langle \phi, A^- e_i \rangle + k_1 \frac{\mathbf{l}_i}{\mu} + k_2 \lambda(t) + k_3 \mu \mathbf{I}_i,$$

$$H_1(\phi, t, e_i) = \lambda(t)[\langle \phi, A^+ e_i \rangle - k_2 \mathbf{1}_i].$$

Functions ϕ^i can be found from the system of equations

$$\frac{d\phi^i(t)}{dt} =$$
$$- \min\{\min_\mu H_0(\phi, t, \mu, e_i), \min_\mu H_0(\phi(t), t, \mu, e_i) +$$
$$H_1(\phi(t), t, e_i)\}, \quad \phi^i(T) = \phi_0^i, \tag{11}$$

and the optimal control $\mu(t, e_i)$ is equal

$$\mu(t,e_i) = \begin{cases} \sqrt{\frac{a}{b}} \text{ if } \sqrt{\frac{a}{b}} \in [\underline{\mu}, \overline{\mu}], \quad b > 0, \\ \underline{\mu} \quad \text{ if } \sqrt{\frac{a}{b}} < \underline{\mu}, \quad b > 0, \\ \overline{\mu} \quad \text{ if } \sqrt{\frac{a}{b}} > \overline{\mu}, \quad b > 0, \\ \overline{\mu} \quad \text{ if } b \le 0, \end{cases} \tag{12}$$

where

$$a(t,e_i) = k_1 \mathbf{l}_i \ge 0, \quad b(t,e_i) = \langle \phi(t), A^- e_i \rangle + k_3 \mathbf{I}_i.$$

The optimal control $u(t,e_i)$ is calculated with the aid of relation

$$u(t,e_i) = \begin{cases} 1, & \text{if} \quad H_1(\phi(t),t,e_i) \le 0, \\ 0, & \text{if} \quad H_1(\phi(t),t,e_i) > 0. \end{cases} \tag{13}$$

Here,

$$H_0(\phi,t,\mu,e_i) =$$

$$\begin{cases} k_2\lambda(t) & for \qquad i=0, \\ \mu(\phi^{i-1}-\phi^i) + \frac{k_1 i}{\mu} + k_2\lambda(t) + k_3\mu & for \ 0<i<N, \\ \mu(\phi^{N-1}-\phi^N) + \frac{k_1 N}{\mu} + k_2\lambda(t) + k_3\mu & for \qquad i=N, \end{cases} \tag{14}$$

and

$$H_1(\phi,t,e_i) = \begin{cases} \lambda(t)(-\phi^0+\phi^1-k_2) & for \qquad i=0, \\ \lambda(t)(-\phi^i+\phi^{i+1}-k_2) & for \ 0<i<N, \\ 0 & for \qquad i=N. \end{cases} \tag{15}$$

Remark 3.1. Notice that for each state $e_i \in S$ the optimal control can be chosen as Borelean measurable function which coincides with (12),(13) almost everywhere in $[0,T]$. Moreover, the number of the state changes is finite with $P=1$ and therefore, the composition $u(t,X_t)$ is progressively measurable, and there exists a predictable version of this control. Full details can be found in Wan & Davis [1979].

4. EXAMPLES

Example 1. Access control.

In order to illustrate these results we consider the example of queuing system with buffer of length $N=2$. Other parameters are the following:

$$\lambda(t) = 1.5 + 0.5\cos 2t,$$
$$\mu = 1, \quad T=10, \quad k_1 = 0.17, \quad k_2 = 0.5.$$

So in order to calculate the value function $V(t,X) = \langle \phi(t), X \rangle$ we have to solve the system of ordinary differential equations (11) for $\phi^i(t)$, $i=0,1,2$ with functions H_0, H_1 given by relations (14), (15).

It is evident that at the state $X=e_2$ the access is impossible, and $u(t,e_2)=0$. At the state $X=e_0$ we get $u(t,e_0)=1$, so if the system is free any job will be accepted. The nontrivial case is $X=e_1$ when there is only one job in queue. In this case the decision depend on time and the future evolution of the rate of input flow. The results of calculations are presented below in Figures 1, 2 for functions $\phi^i(T-t)$ and for control $u(T-t,e_0), u(T-t,e_1)$. Since in this case the Hamiltonian is affine in u the optimal control has a threshold form.

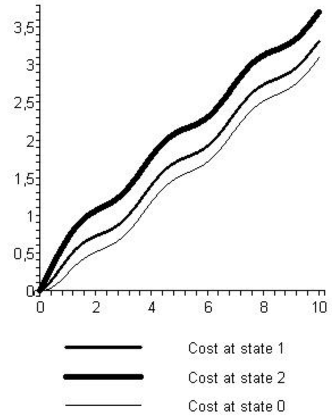

Fig. 1. Cost functions $\phi_i(T-t)$ for $i=0,1,2$. Here $T=10$, and $t \in [0,T]$.

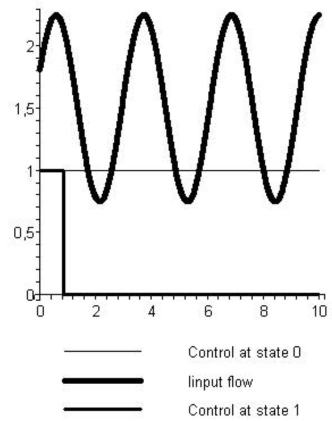

Fig. 2. Control functions $u_i(T-t)$ for $i=0,1$, $u_2(t) \equiv 0$. Here $T=10$, and $t \in [0,T]$.

Example 2. Joint access and the service rate control.

Numerical modelling is performed for the case

$$k_1 = 0.17, \quad k_2 = 0.25, \quad k_3 = 1, \quad T=10$$
$$\mu \in [1.0, 2.0], \quad \lambda(t) = 0.6 + 0.5\cos 2t, \quad \phi_0^0 = 5,$$
$$\phi_0^1 = \phi_0^2 = 0.$$

So we introduce the penalization of the terminal state. The results are presented below in Figures 3, 4, 5.

5. CONCLUSIONS

So we apply the general optimal control setting to various classes of problems arising in theory of queuing systems. However, for any criterion os a type (4) one can calculate also the cost function for any type of controls $u(t,X)$ and thereby one can construct an effective numerical procedure

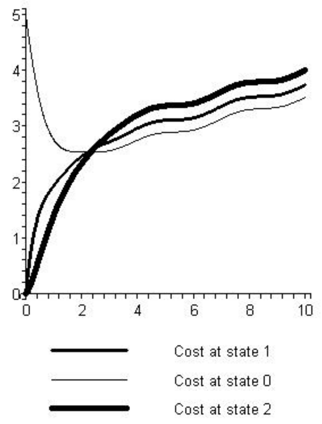

Fig. 3. Cost functions $\phi_i(T-t)$ for $i = 0, 1, 2$. Here $T = 10$, and $t \in [0, T]$.

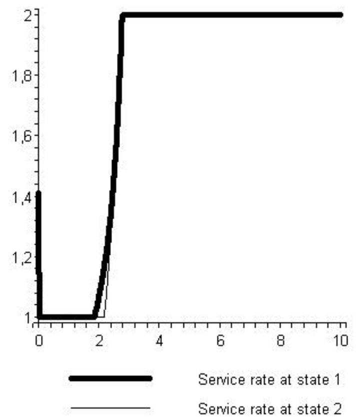

Fig. 4. Service rates $\mu_i(T-t)$, for $i = 1, 2$. Here $T = 10$, and $t \in [0, T]$.

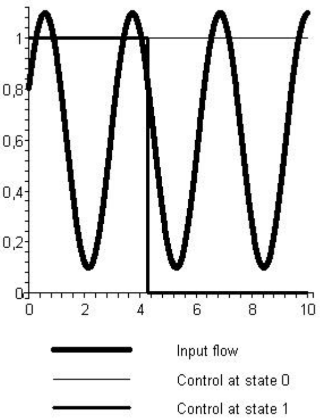

Fig. 5. Controls $u_i(T-t)$, for $i = 0, 1$, $u_2(t) \equiv 0$. Here $T = 10$, and $t \in [0, T]$.

for the solution of the problems with constraints, represented in the form of inequalities with the criteria of a type (4).

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