Dynamic Response of Pedestrian Bridges for Random Crowd-Loading

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Summary

The recent publicity regarding the 18-month closure of London's Millennium Bridge due to excessive lateral vibration response under crowd loading during its opening ceremony has highlighted the necessity for further investigation into the sources of this problem. Current design guidelines focus on single pedestrian dynamic loading and subsequently underestimate the dynamic response associated with crowd loading in the design of pedestrian bridges. This deficiency is addressed in this paper with the mathematical incorporation of random crowd effects into the dynamic analysis procedure. The introduction of a crowd factor (C_f) allows the individual response to be extended to incorporate multiple pedestrians with random arrival times. A subsequent statistical analysis into the mean, variance and distribution shape of C_f allowed the mathematical derivation of an equation stipulating its maximum upper value for a deemed appropriate level of confidence.

Notation

A	= plan area of bridge (m^2)
C	= $\sum_{i=1}^{n} \cos(\varphi_i)$
C_f	= Crowd factor
f (Hz)	= Pacing frequency
F_{lat} (Hz)	= Horizontal fundamental frequency
F_{vert} (Hz)	= Vertical fundamental frequency
L (m)	= Stride length
n	= total number of pedestrians on the bridge
S	= $\sum_{i=1}^{n} \sin(\varphi_i)$
t_m	= time at peak acceleration response
V (m/s)	= Forward speed
v	= dynamic displacement response of a lightly damped pedestrian bridge
$v_{\max}$	= peak displacement response
$\dot{v}_{\max}$	= peak velocity response
$\ddot{v}_{\max}$	= peak acceleration response
α	= maximum practical crowd density (persons/m^2)
φ_i, φ_j	= random phase angles
μ	= mean of the distribution
ρ	= amplitude of v(t)
ω	= frequency of v(t)

1. Introduction

The recent publicity regarding the 18 month closure of London's Millennium Bridge due to excessive lateral vibration response under crowd loading during its opening ceremony has highlighted the necessity for further investigation into the sources of this problem in addition to the requirement for improvement of engineering assessment criteria related to this topic.

The paper identifies three specific areas where current procedures fail to adequately address crowd-loading effects and subsequently endeavours to develop theoretical solutions in response to these deficiencies. The areas are: 1) accurate estimation of the net dynamic effects of a crowd of individuals walking in random phase; 2) recognition of the relationship between walking characteristics and crowd density; and 3) development of assessment criteria to determine whether the potential for the initiation of pedestrian 'lock in' exists. The paper addresses both vertical and lateral vibrations with a focus on the respective fundamental modes.

2. Current design procedures

The current British code BS5400 approaches the dynamic analysis of structures in response to pedestrian loading in terms of peak acceleration response to a single pedestrian load. The technical library of the Australian Institute of Steel suggests the use of the 1997 design guide produced by Murray et al.[1] for the American and Canadian Institutes of Steel Construction. This design guide is relatively comprehensive for building floor systems and has been adapted to include vertical vibration of footbridges. Subsequently, no guidelines for lateral vibration design are provided. Furthermore, only single pedestrian dynamic response has been considered. The only extension to incorporate crowd effects as opposed to individual effects in the vertical load case is with respect to building floor systems, which bear questionable relevance to pedestrian bridges. The failure to address both crowd effects and the lateral dynamic case is a significant design fallacy as exemplified by the case of London's Millennium Bridge.

3. Experimental research on dynamic crowd loading

Quantitative research on the dynamic response of pedestrian bridges to crowd loading is uncommon. There has historically been significantly more research conducted on the dynamic response of building floor systems to crowd loading which has tempted engineers to accept the results of these studies for use in the dynamic analysis of pedestrian bridges. Some of this has questionable relevance for footbridges.

Ellingworth and Tallin[2] considered floor vibrations generated by pedestrian movement. Their results indicated that dynamic forces from groups of people walking about the floor at random would seldom cause serviceability problems because the excitation would lack coherence unless the group was walking in step and thus the dynamic motion would be small.[2] Ellis[3] performed group walking tests on a building floor system and found that the peak acceleration resulting from groups of size up to 32 was approximately twice the corresponding result produced by a 75 kg individual. It was subsequently suggested a factor of 2 could be added to approximate the resultant acceleration for groups up to this size.

These conclusions have questionable relevance for footbridges due to the implicit assumptions in the analyses. Firstly, the fundamental frequency of vibration (f_n) in building floor systems is significantly higher than even relatively short span pedestrian bridges. Murray et al.[1] claim the majority of steel framed floor systems in North America have fundamental frequency in the range 5-9 Hz and suggest most European systems are greater than 9 Hz. The floor examined in the Ellis[3] investigation had a fundamental frequency of 9.43 Hz corresponding to the fifth Fourier component of walking force. This property immediately eliminates the possibility of the fundamental frequency of vibration in a typical floor system being excited by dynamic forces associated with typical pedestrian walking. Conversely, numerous examples exist where the fundamental frequency of vibration of a pedestrian bridge is in the range of typical walking frequencies. Two prominent examples of this are the Jatujak Bridge[4] in Bangkok with a vertical frequency in the fundamental mode of 2.00 Hz and the Millennium Bridge in London with a lateral frequency in the fundamental mode of 0.9 Hz.[5] The Fourier coefficients decline in magnitude with increasing Fourier component and hence the resulting response to walking excitation is significantly less for floors, than for footbridges where the first or fundamental Fourier component can be excited. [6]

The second problem with accepting experimental research conducted on building floors as applicable to pedestrian bridges stems from the inherent geometric difference between the two and the resulting type of excitation induced by pedestrians. Floors are generally "open" areas and hence crowd velocity will often lack synchronisation of direction. Furthermore, there is also a reduced probability of correlation of walking characteristics between occupants due to the presence of obstacles such as columns, walls, partitions and furniture which hamper sustained harmonic walking motion by inducing step breaks and momentum changes. The lack of similar pedestrian objectives results in a reduced likelihood of correlation of pedestrian motion. Conversely, footbridge pedestrians often share the primary walking objective of traversing the bridge with maximum efficiency and in minimum timeframe. Furthermore, motion is restricted to two general directions, with the probability of large crowds travelling in one particular direction quite large in some cases. For instance, a pedestrian bridge linking a stadium to a transport centre is likely to experience significant pedestrian flow rate in one direction at the conclusion of an event. These two factors which are specifically relevant for footbridges result in the alignment of pedestrian objectives which is likely to increase the correlation of at least some characteristics between individual pedestrian motions among a crowd.

Finally, building floor systems are not susceptible to lateral vibrations. This is not true for pedestrian bridges as no lateral restraint is typically provided along the bridge span. The technical documentation describing large lateral vibrations in the Millennium Bridge in 1999 and a Japanese pedestrian bridge in 1993 illustrates this vulnerability.

4. Human walking characteristics

Results of a study[7] investigated the typical vertical and lateral frequency of human walking motion is presented in Table 1.

Table 1: Typical vertical and lateral frequency of human walking motion [7]

	Pacing frequency f (Hz)	Forward speed V (m/s)	Stride length L (m)	Vertical fundamental frequency F_{vert} (Hz)	Horizontal fundamental frequency F_{lat} (Hz)
Slow walk	1.7	1.1	0.60	1.7	0.85
Normal Walk	2.0	1.5	0.75	2.0	1.0
Fast Walk	2.3	2.2	1.00	2.3	1.15
Slow running (jogging)	2.5	3.3	1.30	2.5	1.25
Fast running (sprinting)	>3.2	5.5	1.75	>3.2	>1.6

4.1 Randomness of crowd motion

Ebrahimpour et al.[8] measured dynamic forces generated by small crowds of 2-4 people on a specifically designed platform of 14.2 m span. The observation was made that subjects consciously made adjustments to their footfall frequency and phase as governed by the movement of those around them, thus indicating that people in small crowds tend towards being in phase.[8] There are significant implications here regarding the validity of the assumption of randomness of walking motion in small groups. This observation is supported by Arup's analysis of London's Millennium Bridge, which inferred that the probability of phase synchronisation between pedestrians is as high as 0.3 where there is zero bridge response.[5] It is unclear how this applies to a larger group of more than four members or to a crowd consisting of hundreds or thousands of subjects although intuitively it would seem less valid with increasing group size without some form of organised coordination.

4.2 Previous attempts to predict level crowd synchronisation

Crowd loading has conventionally been considered to have little or no *net* dynamic effect on a pedestrian bridge as there was considered to be effectively zero correlation between walking frequency and phase between the individuals in the crowd. Two previous attempts to predict the level of random crowd walking synchronisation have adopted the approximation of $\sqrt{n}$, where n = total number of pedestrians on the bridge.[9,4] The authors intends to mathematically show that use of this approximation often results in an underestimation of the amount of people in phase synchronisation. Fujino[9] proved with the use of practical data that the level of synchronisation on a pedestrian bridge with 2000 capacity in Japan was in the range of 0.2n rather than $\sqrt{n}$ giving a discrepancy of 800%.

5. "Lock-in"

'Lock in' can be defined as *where pedestrians adjust their walking phase and frequency to match that of the bridge response in order to maximise the predictability of their interaction with the bridge and to maintain balance.*[10]

A 1972 report cited a case where a recently constructed steel footbridge had experienced strong lateral vibration when subjected to crowd loading comprising 300-400 people.[11] It explained how the shift in a pedestrian's centre of gravity occurs at a frequency equal to half the pacing rate and exerts a horizontal force on the underlying bridge deck. They reasoned "it could be supposed that in this case pedestrians synchronised their step with the bridge vibration, thereby enhancing the bridge vibration considerably".[11(p.2)]

Bachmann and Ammann[7] recorded from experimentation that particular difficulty was encountered by pedestrians in relation to maintaining step when disturbed by vertical vibration displacements of 10-20 mm and that under such response a person adjusts to synchronise his/her walking characteristics with the structure response. This discovery was dismissed by Ellis[3] with the assertion that this amount of dynamic displacement is far greater than normally encountered with floors. However, it would not be unlikely for some longer span pedestrian bridges to experience this magnitude of dynamic response under crowd loading particularly with the consideration that vibration of 10-20 mm corresponds to amplitude of 5-10 mm.

A subsequent investigation on a large Japanese cable-stayed footbridge linking a boat race stadium and bus terminal made several noteworthy observations with respect to walking motion exhibited by the crowd of maximum size 2000. Peak amplitude of lateral vibration of 10 mm was observed which the authors reported could be felt and subsequently resulted in interruption of their walking motion.[9] Furthermore, pedestrians were overheard remarking on the vibration and some reached for the handrails while walking.[9] This is an undesirable situation as it clearly represents a failure under serviceability design criteria.

The investigation into the causes of the large discrepancy between calculated and actual amplitude supported the pedestrian 'lock-in' theory and found that this had subsequently resulted in a gross underestimation of the number of people in phase with the bridge natural frequency. The report explained; "first a small lateral random motion is induced by random human walking forces, and walking of some pedestrians is synchronised to the girder motion. Then resonant force acts on the girder and consequently, girder motion is increased. Walking of more pedestrians is synchronised, increasing the lateral girder motion".[9 (p. 757)] This paper[9] provides a clear method for determining the initial *random* motion described in the previous passage.

Pedestrian 'lock-in' between individuals in a crowd was a principal cause of the excessive lateral motions observed at the opening of the Millennium Bridge in London.[10]

5.1 Implications of 'lock in'

'Lock-in' essentially ensures that footfall forces are applied in the range of the resonant frequency of the bridge and moreover that the phase angles of these forces are synchronised between pedestrians. As the amplitude of the motion increases, the degree of phase correlation between individuals rises, subsequently increasing the applied dynamic forces. The vibration is of a self-excited nature and that a snowball effect eventuates as the increase in dynamic forces produces further magnification of the amplitude. It is logical to assume that the serviceability criteria of human comfort will automatically be exceeded if the 'lock-in' sequence is initiated. Although the Fujino[9] and the Millennium Bridge cases are primarily concerned with lateral 'lock in', it is important to examine both the vertical and lateral load cases as evidence supports its occurrence in response to either mode of excitation.

6. Theoretical incorporation of Crowd Effects into Bridge Response

This paper focuses on the development of 3 principal areas of the dynamic analysis procedure. These areas all relate to the accurate incorporation of crowd effects into the dynamic analysis. The method for determination of the dynamic structural response of a bridge subject to the harmonic load produced by a single pedestrian is well known. Similarly, the evaluation of dynamic bridge response with respect to human comfort criteria is a widely practised conventional analysis. These procedures will be drawn upon but will not form the focal points of the study presented in this paper. The 3 key objectives of this paper are:

1) The accurate extension of theory to allow estimation of the likely dynamic bridge response to crowd loading;
2) The recognition of the relationship between walking characteristics and crowd density; and
3) The addition of a step in the dynamic analysis which entails evaluation of the propensity for pedestrian 'lock in' to occur

6.1. Incorporation of random crowd effects

The following section investigates the effect of excitation resulting from multiple pedestrians with equal walking frequencies and random phase angles as determined by arrival time.

The dynamic response of a lightly damped pedestrian bridge under the harmonic load of a single pedestrian is:

$$v(t) = \rho \sin(\omega t) \tag{1}$$

Where ω and ρ are frequency and amplitude of the dynamic response.

The dynamic response resulting from n pedestrians, with random arrival times modelled by introducing random phase angles (φ_i), can be represented by:

$$v(t) = \rho \sum_{i=1}^{n} \sin(\omega t + \varphi_i) \tag{2}$$

By defining:

$$S = \sum_{i=1}^{n} \sin(\varphi_i) \qquad \text{and} \qquad C = \sum_{i=1}^{n} \cos(\varphi_i) \tag{3}$$

The Equation (2) can be rewritten as:

$$v(t) = \rho \sin(\omega t).C + \rho \cos(\omega t).S \tag{4}$$

Velocity and acceleration responses can be obtained with differentiation of Equation (4).

Peak acceleration response at time t_m:

$$\frac{d\ddot{v}}{dt} = -\rho\omega^3 \cos(\omega t).C + \rho\omega^3 \sin(\omega t).S = 0 \tag{5}$$

$$\therefore \tan(\omega t_m) = \frac{C}{S} \tag{6}$$

Now:

$$\sin(\omega t_m) = \frac{\tan(\omega t_m)}{\sqrt{1+\tan^2(\omega t_m)}} = \frac{\frac{C}{S}}{\sqrt{1+\frac{C^2}{S^2}}} = \frac{C}{\sqrt{C^2+S^2}} \tag{7}$$

and

$$\cos(\omega t_m) = \frac{1}{\sqrt{1+\tan^2(\omega t_m)}} = \frac{1}{\sqrt{1+\frac{C^2}{S^2}}} = \frac{S}{\sqrt{C^2+S^2}} \tag{8}$$

$$\therefore \ddot{v}_{\max}(t) = -\rho C\omega^2 \frac{C}{\sqrt{C^2+S^2}} - \rho S\omega^2 \frac{S}{\sqrt{C^2+S^2}} = -\rho\omega^2 \left(\frac{1}{\sqrt{C^2+S^2}}\right)\left(C^2+S^2\right)$$

$$\therefore \ddot{v}_{\max} = -\rho\omega^2 \sqrt{\left(C^2+S^2\right)} \tag{9}$$

Similarly for peak displacement and velocity of motion:

$$\therefore v_{\max} = -\rho\sqrt{\left(C^2 + S^2\right)} \tag{10}$$

$$\therefore \dot{v}_{\max} = -\rho\omega\sqrt{\left(C^2 + S^2\right)} \tag{11}$$

6.2 Crowd factor (C_f)

Equation (11) dictates that the response to a crowd of *n* people is given by the response to a single pedestrian multiplied by the factor $\sqrt{\left(C^2 + S^2\right)}$. This factor will now be defined as the "crowd factor" and given the symbol "C_f" for the purposes of this paper. Note that the crowd factor is also applicable for determination of peak displacement and peak velocity response under crowd loading. The crowd factor allows the response of a pedestrian bridge to crowd loading to be determined by simulating the amount of pedestrians who are walking in phase and hence dynamically exciting the bridge together.

6.3 Statistical investigation of C_f

The crowd factor is a function of crowd size, *n*. However, its value is not constant for one particular bridge even once crowd size has been determined. This is due to its dependence on the relative arrival times of pedestrians in the crowd, which vary randomly each time a crowd of size *n* is passed over the bridge. This draws attention to the requirement for knowledge relating to the distribution shape, mean and variance of C_f. This will enable calculation of the maximum reasonable value of the crowd factor with a deemed appropriate level of confidence for engineering application.

$$C_f = \sqrt{\left(\sum_{i=1}^{n} \sin\varphi_i\right)^2 + \left(\sum_{i=1}^{n} \cos\varphi_i\right)^2} \tag{12}$$

Note that C_f^2 will be examined and relevant adjustments made at the conclusion for simplicity of mathematics.

6.3.1 Mean and variance of Cf

Rearranging the expression in Equation (12) gives:

$$C_f^{\,2} = (\sum \sin\phi_i)^2 + (\sum \cos\phi_i)^2 = \sum (\sin^2\phi_i + \cos^2\phi_i) + 2\sum_{i<j} (\sin\phi_i \sin\phi_j + \cos\phi_i \cos\phi_j)$$
$$= n + 2\sum_{i<j} \cos(\phi_i - \phi_j). \tag{13}$$

If phase angle (φ_i) is an independent random angle in the range (-π, π), then:

$$E(\cos(\phi_i - \phi_j)) = 0 \tag{14}$$

and

$$Var\big(2\cos(\phi_i - \phi_j)\big) = E\big(2\cos(\phi_i - \phi_j)\big)^2 - \mu^2 \tag{15}$$

As $\mu\left[2\cos(\phi_i - \phi_j)\right] = 0$, the variance becomes:

$$Var(2\cos(\phi_i - \phi_j)) = E(4\cos^2(\phi_i - \phi_j)) = 4E(\frac{1}{2}(1 + \cos^2(\phi_i - \phi_j))) = 2 \tag{16}$$

Since there are *n choose 2* $\left(= \frac{n(n-1))}{2} \right)$ terms in the sum, the sum of the variances is *n(n-1)*. Possible correlations between the terms in the sum must also be taken into account, but since

$$E(\cos(\phi_i - \phi_j)\cos(\phi_k - \phi_l)) = 0 \tag{17}$$

Unless *i = k , j = l,* the correlations are all 0. Therefore if ϕ_i is uniformly distributed in the range (-π, π) and *i* is independent then:

$C_f^{\,2} = (\sum \sin\phi_i)^2 + (\sum \cos\phi_i)^2$ has mean *n* and variance *n.(n-1)*.

This result is in accordance with prior studies which have indicated the number of pedestrians in a crowd who are walking in phase is equal to $\sqrt{n}$.[4,9] Note the above result is for C_f^2 and hence C_f has expected mean $\sqrt{n}$.

6.3.2 Distribution shape

$$E(\sin\varphi_i) = 0 \tag{19}$$

$$\therefore Var(\sin\varphi_i) = E(\sin^2\varphi_i) = E\{\frac{1}{2}(1 - \cos 2\varphi)\} = \frac{1}{2} \Rightarrow Var(\sin\phi_i) = \frac{1}{2} \tag{20}$$

Thus by the central limit theorem $\sum_{i=1}^{n} \sin\phi_i \sim N\left(0, \frac{n}{2}\right)$ then by scaling:

$$\sqrt{\frac{2}{n}} \sum_{i=1}^{n} \sin\phi_i \sim N(0,1) \; approx. \Rightarrow \frac{2}{n} (\sum_{i=1}^{n} \sin\phi_i)^2 \sim \chi_1^2 \; approx.$$

The term can be approximated with a χ_1^2 distribution.

If the other term, $\sum_{i=1}^{n} \cos\phi_i$, were independent of $\sum_{i=1}^{n} \sin\phi_i$ then we could add the two χ_1^2 variables together to give a χ_2^2 distribution.

The independence of $\sum_{i=1}^{n} \cos\phi_i$ and $\sum_{i=1}^{n} \sin\phi_i$ can be explored with calculation of the covariance between the terms:

$E\left((\sum \sin\phi_i)^2 (\sum \cos\phi_i)^2\right)$. The only terms that survive are

$$E(\sum \sin^2\phi_i \cos^2\phi_i) = nE\{\frac{1}{8}(1 - \cos 4\phi)\} = \frac{n}{8} \tag{21}$$

and

$$E(\sum \sin^2\phi_i \cos^2\phi_j) = n(n-1)[E\{\frac{1}{2}(1 - \cos 2\phi_i)\}E\{\frac{1}{2}(1 + \cos 2\phi_j)\}] = \frac{n(n-1)}{4}. \tag{22}$$

Now $E(\sum \sin\phi_i)^2 = E(\sum \cos\phi_i)^2 = \frac{n}{2}$, so

$$Cov\left[\frac{2}{n}(\sum_{i=1}^{n} \sin\phi_i)^2 \frac{2}{n}(\sum_{i=1}^{n} \cos\phi_i)^2\right] = \frac{4}{n^2}\left[\frac{n(n-1)}{4} + \frac{n}{8} - \left(\frac{n}{2}\right)^2\right] = -\frac{1}{2n}. \tag{23}$$

Since the covariance of the two terms tends to 0 as $n \to \infty$, we might expect that

$$\frac{1}{n}\{(\sum \sin\phi_i)^2 + (\sum \cos\phi_i)^2\} \sim \exp(1) \; approx.$$

Since $X/2 \sim \exp(1)$ if $X \sim \chi_2^2$ the above equation can be approximated as

$$(\sum \sin\phi_i)^2 + (\sum \cos\phi_i)^2 \sim \exp(n)$$

A simulation of C_f^2 produced the histogram outlined in Figure 1.

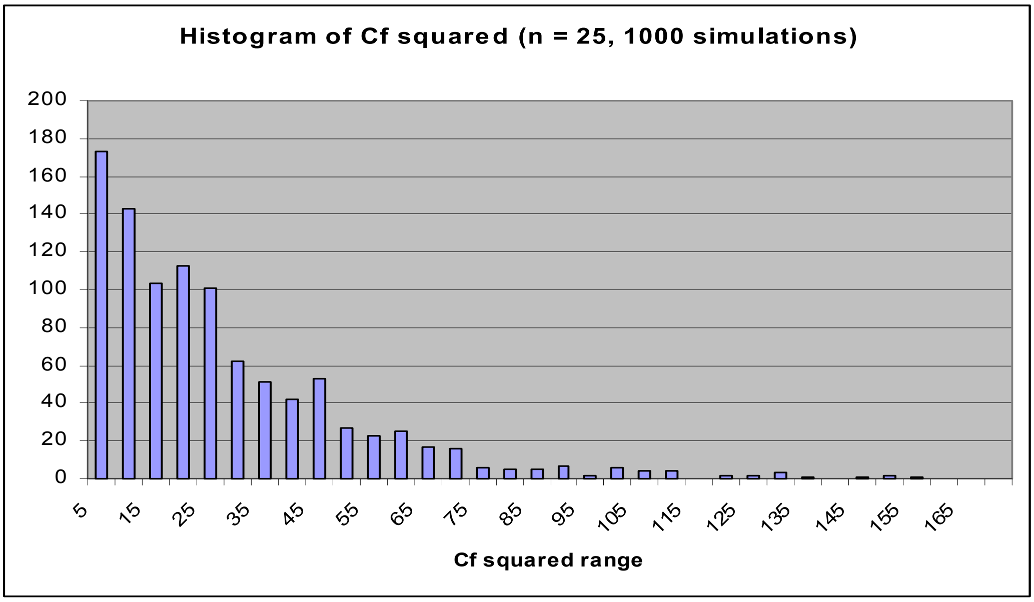

Figure 1: Histogram of 1000 simulations of C_f^2 for n = 25

The data has mean and variance of 25.29 and 596.3 respectively which closely resemble the respective theoretical results *n* (= 25) and *n.(n-1)* (= 600) as previously derived.

The best-known distribution that has the same mean and standard deviation is the exponential of the form:

$$f(x) = \frac{1}{\mu} e^{-x/\mu}, x \geq 0. \quad (24)$$

Where μ is the distribution mean = *n* for C_f^2.

Execution of an ascending sort of the 1000 C_f^2 simulations and then plotting against 1000 sorted points from an exponential distribution of mean 25 results in the plot illustrated in Figure 2.

Figure 2 highlights the accuracy of the aforementioned exponential distribution in approximating the distribution of C_f^2. There is only one significant outlier in 1000 simulations. The accuracy of fit appears to slightly decrease for the upper range values of C_f^2 although some random variation is to be expected.

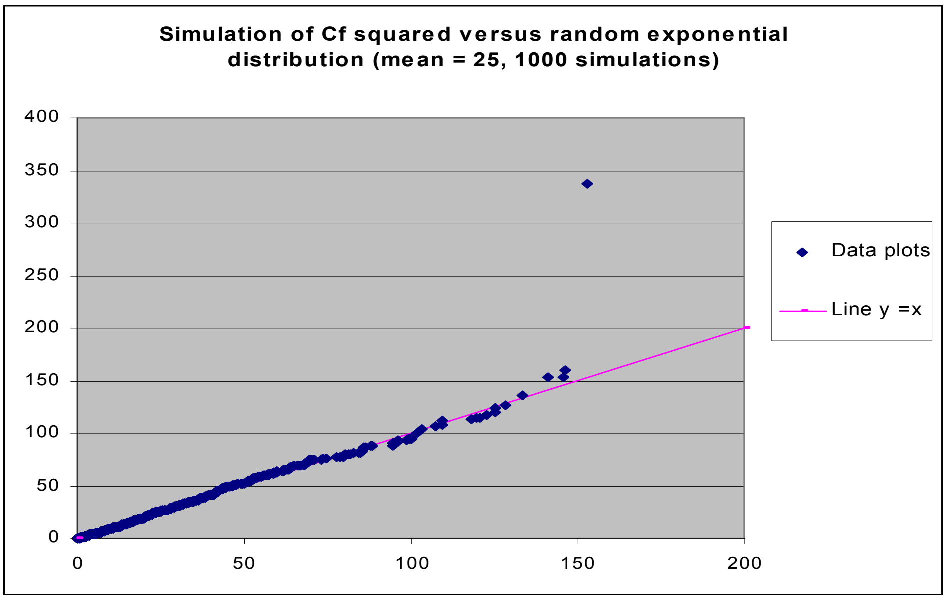

Figure 2: C_f^2 versus exponential distribution points mean 25 (1000 simulations)

6.4 Estimation of maximum C_f

Now the distribution function of C_f^2 has been determined, a critical C_f^2 value can be extracted relating to the boundary of a required upper confidence interval. This will allow the design engineer to determine an upper critical value of C_f which will not be exceeded with a deemed appropriate level of confidence.

The critical value can be obtained through integration of the distribution function $f(x) = \frac{1}{\mu} e^{-x/\mu}$ and equating the result to 1 – L and solving for x where L is the required confidence level expressed as a fraction. This yields:

$$x_L = -\mu \log_e (1 - L)$$

Recognising x = C_f^2 and μ = n:

$$C_{f,L} = \sqrt{-n \log_e (1 - L)} \qquad (25)$$

For example, Equation (25) states that the 95% confident crowd factor ($C_{f,\ 95}$) for a footbridge with a crowd of n = 25 pedestrians is equal to 8.65. The technical interpretation of this is that the net dynamic effect of 25 pedestrians with randomly distributed arrival times and constant footfall frequency will not exceed the dynamic effect of 8.65 perfectly synchronised pedestrians in 95% of cases.

This is a highly valuable result from an engineering perspective as the crowd factor C_f can be estimated to any stipulated confidence level without the requirement for time consuming computer simulation.

6.5 Required confidence level

As the dynamic response of pedestrian bridges is primarily a serviceability problem, the required confidence level for analysis is not a well-defined parameter and engineering judgment must be used to arrive at a conservative yet practical level. The authors suggest a 90% level satisfactorily achieves this balance.

7. Relationship between crowd density and walking characteristics

Bachmann and Ammann[7] found that uninhibited walking was not possible at crowd densities beyond 1.6-1.8 persons/m^2. This has ramifications for the underlying assumptions in the dynamic analysis relating to the characteristics of walking motion of individuals in high-density crowds. This area requires further research, as very little material exists in technical literature relating to this subject.

7.1 Vertical walking characteristics in high density crowds

The fact that uninhibited walking motion is not possible at crowd densities beyond 1.6-1.8 persons/m^2 has important ramifications for the vertical dynamic load case as it is anticipated that the occurrence of inhibited walking motion would significantly decrease the vertical dynamic force per person exerted on the bridge deck by individuals below the assumed 40% static weight per step. The limiting upper crowd density of 1.6-1.8 persons/m^2 should hence be adopted for vertical analysis.

7.2 Lateral walking characteristics in high density crowds

There is a general consensus among technical literature that the magnitude of horizontal force transfer during walking is approximately 4% of static body weight. Most research and virtually all dynamic analysis to this date relating to walking characteristics has been conducted on typical uninhibited walking motion, overlooking the effect that inhibited conditions prevalent in high density crowds have on the walking characteristics of individuals.

One remarkable observation was made in the previously discussed investigation of a Japanese footbridge experiencing lateral vibration under crowd loading conducted by Funjio. In heavily congested periods the amplitude of head lateral motion was significantly larger than for free walking periods even after allowing for superposition of the experienced bridge vibration.[9] The explanation offered was that larger lateral steps are taken when free forward stepping is not possible due to congestion.[9] The author considers this a reasonable inference relating to walking characteristics based on anecdotal experience of pedestrian behaviour in high density flows where pedestrians develop a tendency to 'waddle' or 'rock' from side to side in a more pronounced manner than they otherwise would under unrestricted conditions. This type of walking motion is likely to significantly increase the magnitude of lateral force per person exerted onto the bridge deck. Human movement theory supports this conclusion by stating that lateral amplitude of walking motion increases with decrease in forward walking velocity as illustrated by Figure 3.[12]

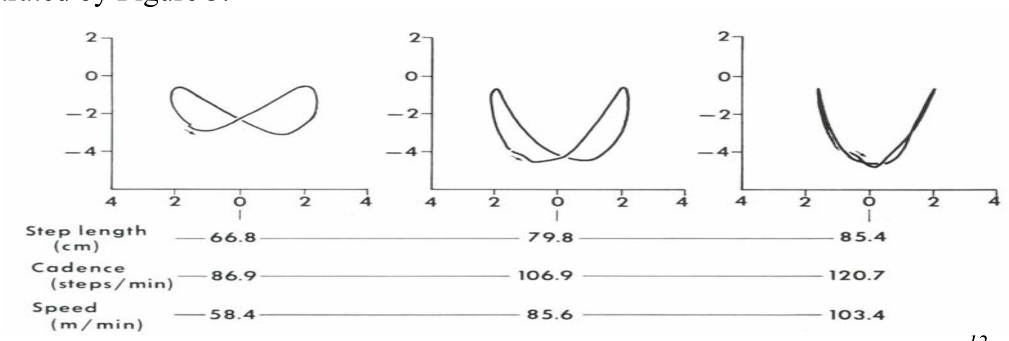

Figure 3 Lateral movement of centre of mass with forward walking velocity[12]

Furthermore, the velocity associated with high-density crowds is likely to be even less than 58 m/min perhaps even further accentuating the amplitude of walking motion. The implication of this is that the limiting state of uninhibited walking motion applicable to the vertical load case is not valid for the lateral case. Instead, high crowd densities causing inhibited walking motion serve to provoke the critical load case under horizontal dynamic analysis. This situation is particularly critical as not only does evidence suggest larger lateral force per person is generated in high density flows but there are more people per square metre than can be allowed for under the assumption of uninhibited walking motion.

There is some uncertainty as to the maximum crowd density, which should be adopted for analysis of the lateral load case but a value of 2.2 persons/m^2 is considered reasonable.

This investigation suggests that a conservative value of lateral force per person under high density crowd flow is in the range of 6% of static body weight in contrast to the currently assumed value of 4% based on uninhibited flow, although the requirement for further quantitative research is evident.

7.3 Summary of crowd density and force per person

Table 2: Suggested maximum crowd density and force transfer per person for vertical and lateral load cases

	Load case	
	Vertical	**Lateral**
Critical load case	Uninhibited walking conditions	Inhibited walking conditions
Maximum crowd density	1.6-1.8 persons/m^2	2.2 persons/m^2
Force per person	40% static weight	6% static weight

8. "Lock-in" assessment criteria

It must be ensured that random dynamic bridge response is not sufficient to initiate the 'lock-in' sequence. There is a general consensus among technical literature that the principal determinant of 'lock in' is amplitude of walkway motion in both the vertical and lateral cases.

Bachmann and Ammann[7] experimentally determined that pedestrian walking characteristics were synchronised with the structural response when disturbed by vertical vibration of 10-20 mm. Experimental analysis on laterally vibrating floors also concluded that walking frequency and phase becomes synchronised to the floor when lateral vibration reaches 10-20 mm.[13] Dallard et al.[10(p. 14)] claim that 'lock in' can initiate when amplitude of walkway motion is only "a few millimetres". A non conservative estimate of 5 mm amplitude will be suggested for use as the criteria for initiation of vertical or lateral pedestrian 'lock-in'. Note 5 mm amplitude corresponds to 10 mm vibration. Further research into the accuracy of these values would be beneficial.

8.1 Suggested evaluation criteria

Table 3 provides a summary of evaluation criteria for assessment of bridge response under crowd loading based on information extracted from technical literature.

Table 3: Summary of suggested evaluation criteria for the analysis

		Limiting value	
Criteria	**Determinant parameter**	**Vertical**	**Lateral**
Human comfort	Peak acceleration	7.5% g	2 – 4% g
Lock in	Peak amplitude	5mm	5mm

9. Determination of *n*

$$n = \alpha(0.5A) \tag{26}$$

Where α = maximum practical crowd density (persons/m^2) and A = plan area of bridge (m^2). The 0.5 area reduction factor simulates the lumping of half total mass at midspan for the fundamental mode. Crowd density (α) requires a clear distinction between the vertical (1.6 persons/m^2) and lateral (2.2 persons/m^2) load cases as previously discussed.

10. Conclusions

The paper contended that current design procedures are based on single pedestrian loading and subsequently underestimate the magnitude of dynamic forces associated with crowd loading in the analysis. This was supported by evidence of historical cases where dynamic response of pedestrian bridges to crowd loading was unsatisfactory. The difficulty in application of results of these studies to pedestrian bridges was outlined due to the implicit differences between building floor systems and pedestrian bridge decks. Finally, technical literature relating to a phenomenon known as 'lock in' was examined and its significant implications were discussed.

The second section of this paper comprised of a theoretical development of engineering procedures with a focus on the incorporation of random crowd effects as opposed to individual pedestrian loading into the dynamic analysis. The paper separated this general aim into three specific objectives consisting of:

1) The accurate estimation of the net dynamic effects of a crowd of individuals walking in random phase
2) Recognition of the relationship between walking characteristics and crowd density and
3) Incorporation of an assessment of the potential for pedestrian 'lock in' to occur into the dynamic analysis.

The introduction of the crowd factor allowed the theoretical extension of theory based on single pedestrian dynamic response to incorporate the effects of multiple pedestrians with random phase angles. A highly valuable engineering result was obtained with the statistical derivation of an equation dictating the maximum value of the crowd factor with a deemed appropriate level of confidence: $C_{f,L} = \sqrt{-n \log_e (1-L)}$

The investigation into the relationship between walking characteristics and crowd density used human movement theory to prove that lateral force per person increases under inhibited walking conditions prevalent in high crowd densities. This result calls into question the accuracy of conventional lateral dynamic analyses under crowd loading which base estimates of lateral force per person based on uninhibited walking conditions.

The requirement for inclusion of assessment of the potential for pedestrian 'lock in' was addressed with the introduction of an additional step in the dynamic analysis procedure. The results and observations of technical literature were used to determine that the primary determinant of 'lock in' is excessive amplitude of vibration. Furthermore, these sources estimated that the critical amplitude, which initiates the 'lock in' sequence, was in the range of 5 mm for both the lateral and vertical load cases.

11. Acknowledgements

Authors are grateful to Dr Geoff Taplin, Maunsell Australia, for his valuable suggestions and comments for this research project.

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